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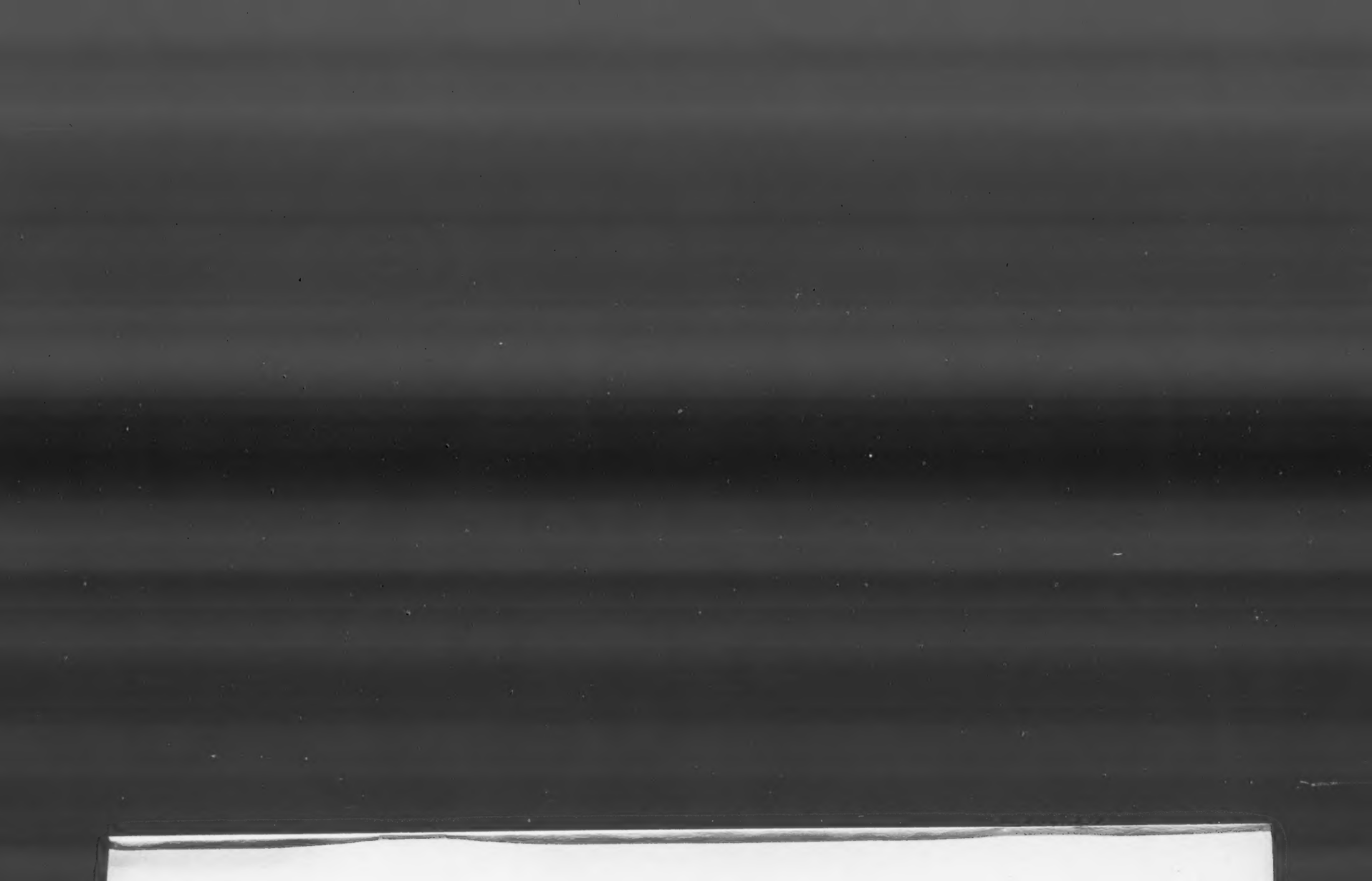

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APPROXIMATE GAMMA RAY FLUX CALCULATIONS OUTSIDE A REACTOR CORE

by

MATTS ROOS

ACTA POLYTECHNICA SCANDINAVICA

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INTRODUCTION

The core of a nuclear reactor constitutes an intense source of gamma radiation which gives rise to serious shielding and heating problems. To solve these problems one needs reliable methods for calculating the gamma ray flux and dose rate outside the reactor core.

It is, however, not generally possible to solve gamma ray problems exactly, although the differential equations can be formulated rather easily and the physical constants are fairly well known. Even simple problems tend to overtax the capabilities of the largest computing machines and practical problems can only be dealt with approximatively after strong simplifications.

In this work we investigate the effects of various approximations which permit the calculation of gamma radiation from a reactor core by manual methods. We consider a heterogeneous thermal reactor with cylindrical rod fuel elements forming a cylindrical core. A reactor of this type is the early Swedish R3b project, which forms the basis for our calculations.

We eventually conclude, that one of the methods is best suitable for computing machine programming. Such a program has in fact been described in a report by the author (1959 b).

1. BASIC DATA OF THE R3b PROJECT

The R3b is a D_2O-cooled and D_2O-moderated natural uranium reactor, designed to be operated at 94 MW power. The 112 fuel elements are arranged on a square lattice with a 25 cm pitch so that they form an approximately cylindrical core. Each fuel element consists of 27 cylindrical Zr-canned UO_2 rods enclosed in a Zr pressure tube. The material proportions of the core and the main dimensions are listed in table 1.

Table 1.

Basic data of the R3b project.

Diameter of the core ($2R_o$)	319	cm
Extrapolated diameter of the core (2R)	388.6	cm
Height of the core (H_o)	332	cm
Extrapolated height of the core (H)	379.3	cm
Diameter of fuel element	11.16	cm
Volume of UO_2 in fuel element	$1.118 \cdot 10^4$	cm^3
" " D_2O " " "	$1.267 \cdot 10^4$	cm^3
" " Zr " " "	$0,639.10^4$	cm^3
Mass " UO_2 " core	12.77	t
" " D_2O " "	26.76	t
" " Zr " "	4.65	t
Average density of fuel element	5.230	g/cm^3
" " " core	1.665	g/cm^3
Average power density in the core ($\bar{P}_V$)	3.543	W/cm^3

The gamma sources and energy spectra of reactors of the type considered have recently been discussed in a publication by the author (1959 a). Since we in this work, however, are dealing with manual methods, requiring the energy spectra to be divided into a few rough groups, it is sufficient to use the old spectra of Braun (1957) which also were used in the original report by Braun and Roos (1957). Table 2 lists the data of Braun's spectra necessary in this work. Here $N_{tot}(\bar{E})$ is the total energy release in the energy group of mean energy $\bar{E}$, and $N_{esc}(\bar{E})$ is the part of the total energy which escapes from a fuel element. The fraction $N_{esc}(E)/N_{tot}(E)$ is the escape probability, a function of $r\mu_{en}$, where r is the radius of the fuel element and μ_{en} is the energy absorption coefficient. The escape probability has been tabulated by Placzek & al. (1953).

Table 2.

Energy groups of the spectra of Braun.

Group interval (MeV)	Mean energy $\bar{E}$ (MeV)	$N_{tot}(\bar{E})$ (MeV/f)[x)]	$N_{esc}(\bar{E})$ (MeV/f)[x)]
0 ··· 3	2	17.56	6.08
3 ··· 5	4	5.95	2.62
5 ··· 8	7	1.59	0.62
0 ··· 4	3		7.62
4 ··· 8	6		1.70
0 ··· 8		25.10	9.32

x) We use f as short-hand notation for "fissions".

In practical shielding calculations the gamma ray sources have to be expressed in units of MeV/s rather than in MeV/f, which is convenient for spectral representations. Depending on the geometrical shape of the source one defines a point source strength S (MeV/s), a line source strength S_L (MeV/cm s), a surface or plane source strength S_A (MeV/cm^2 s) or a volume or volumetric source strength S_V (MeV/cm^3 s). For example the average gamma source strength $\bar{S}_V(\bar{E})$ in the energy group with mean energy $\bar{E}$, per unit reactor core volume is defined as

$$\bar{S}_V(\bar{E}) = k\,\bar{P}_V N(\bar{E}), \qquad (1.1)$$

where

$\bar{P}_V$ average power density or average power per unit reactor core volume, (W/cm^3),

$N(\bar{E})$ energy release in the spectral group with mean energy $\bar{E}$, (MeV/f), and

k $3.1.10^{10}$ f/Ws, a constant which relates the number of fissions in a reactor to the energy release. Its numerical value is based on an assumed release of 200 MeV per fission.

In a cylindrical reactor core the thermal neutron flux follows a cosine distribution along the core centerline and a J_o-distribution along a radius from the center of the core. Thus also the reactor power and the gamma ray source strength have the same distribution. The relation between the maximum value, at the center of the core, of a quantity with this distribution, and its average value over the core, can be described as the product of two factors f_r and f_a, the radial and axial form factors, respectively. Their definitions are

$$\frac{1}{f_r} = \frac{1}{n} \sum_{i=1}^{n} J_o\left(\frac{2.405\ r_i}{R}\right), \tag{1.2}$$

$$\frac{1}{f_a} = \frac{2}{H_o} \int_{o}^{H_o/2} \cos\left(\frac{\pi h}{H}\right) dh = \frac{2H}{\pi H_o} \sin\left(\frac{\pi H_o}{2H}\right), \tag{1.3}$$

where

r_i radial coordinate of the i:th fuel element

h axial coordinate, measured from the center of the reactor core

n total number of fuel elements (112 in R3b), and R, H and H_o as defined in table 1. Inserting the numerical values for the R3b, one obtains

f_r = 1.586
f_a = 1.401
$f_r f_a$ = 2.22

In this way the axial source strength distribution of the i:th fuel element, regarded as a line source, can be written

$$S_{Li}(h) = S_{Li}(o) \cos\frac{\pi h}{H}, \tag{1.4}$$

where

$$S_{Li}(o) = \frac{k\ f_r\ f_a\ P\ N(\bar{E})}{n\ H_o} J_o\left(\frac{2.405 r_i}{R}\right), \tag{1.5}$$

and P = total power (94 MW for R3b).

Inserting the numerical values one obtains

$$\frac{k\, f_r\, f_a\, P}{n\, H_o} = 1.74.10^{14}\left(\frac{f}{cm\ s}\right).$$

2. GAMMA ATTENUATION THEORY

The gamma ray flux from a line source, or in general from any extended (line, surface or volume) source, is always computed as a sum or integral over the differential point source elements into which the extended source can be subdivided. Thus the basic radiation problem is to compute the flux from a point source. When this is known the flux from any extended source is only a geometrical and mathematical problem. Simple as it may seem, these problems can in practice seldom be solved, particularly not by manual methods. Throughout this work we must for instance approximate (1.4) by (1.5), a constant, because a line source with a strength distribution like (1.4) leads to unsolvable integrals.

The probability that a photon, which has been emitted from a point source of strength S, is found within unit solid angle $1/r^2$ at a radius r is

$$1/4\pi r^2 . \tag{2.1}$$

The probability that the photon will travel a distance r without collision is

$$e^{-b}, \tag{2.2}$$

where

$$b = \int_0^r \mu(r')\, dr' , \tag{2.3}$$

and $\mu(r')$ is the total macroscopic collision cross-section, also denoted total ab-

sorption coefficient μ_{tot}. In the case of n finite homogeneous shields of thickness t_i and total cross-section μ_i, eq. (2.3) simplifies to

$$b = \sum_{i=1}^{n} \mu_i t_i . \qquad (2.4)$$

The uncollided flux Φ_u at a distance r from the point source, or the so called point kernel, is then defined as the product of the source strength S and the probabilities (2.1) and (2.2)

$$\Phi_u = \frac{S}{4\pi r^2} \cdot e^{-b} \qquad (2.5)$$

If S denotes the number of photons emitted per second, Φ_u is the number flux (photons/cm^{2}s), and $\Phi_u E$ is the energy flux (MeV/cm^{2}s). In the following we shall use a slightly different notation. By S we shall mean the source strength in MeV/s, so that Φ_u is the energy flux.

The conditions under which the real flux Φ equals the uncollided flux Φ_u, are called narrow beam conditions. The reason for this is evident from a consideration of the interaction processes which form the total collision cross-section.

The most important gamma-ray interaction processes out of about a dozen processes are

1) the photoelectric effect,
2) pair production,
3) the Compton effect.

1) In the photoelectric effect the photon transfers all its energy to an electron of the target atom, which thereby is emitted from the atom. Thus the photon is absorbed, and since the electron will not travel far in a shielding material, all the gamma-ray energy can be said to be absorbed in the immediate vicinity of the target atom.

2) In pair production all the energy of the photon is consumed by the creation of an electron and a positron. The same argument holds as in the previous case; thus pair production is also an absorptive process.

3) The Compton effect is essentially a scattering process in which the photon collides with a free electron and then continues its path into a changed direction with reduced energy. Thus the photon never disappears, but one can still represent the Compton effect by the sum of a scattering cross-section and an absorption cross-section, if one by absorption means the average fractional energy reduction in the scattering collision.

The total cross-section μ_{tot} is for historical reasons called total absorption coefficient, which is incorrect since it also includes pure scattering. Another cross-

section in use is the energy absorption coefficient μ_{en}, which equals the total cross-section less the pure scattering cross-section.

Only in a very narrow beam of gamma rays, where even a small angular deviation would bring the photon out of the beam, can a photon disappear entirely by a scattering collision and the real flux be represented by (2.5) (where μ is the total absorption coefficient μ_{tot}).

In the general or broad beam case a considerable number of Compton-scattered photons undergo one or more scattering collisions in a thick shield and thus have a fairly large probability to stay within the beam. This means that the uncollided flux (2.5) always represents an underestimate of the real flux.

In order to find an expression for the total flux Φ in analogy with (2.5) one would have to consider both angular and energetic distributions of the scattered photons in space, but in all geometries save possibly for the simplest (homogeneous one-medium problems) this is a hopeless task, particularly with manual methods. One therefore has chosen to represent the total flux by .

$$\Phi = B\Phi_u \,, \qquad (2.6)$$

where B is called "build-up factor", and is a function of photon energy, shielding medium, thickness of shield (in dimensionless relaxation lengths b) and sometimes angle of incidence on shield. Further B is also a function of the geometry, but the customary build-up factors are always computed for infinite or semi-infinite media or infinite slabs of finite thickness, so that we have no choice of build-up factors for different finite geometries.

Depending on whether the fluxes in (2.6) are number fluxes, energy fluxes, fluxes of absorbed energy, dose rates etc. there are corresponding build-up factors in use. In manual calculations it is customary to use dose build-up factors both for energy fluxes and for dose rates, because the difference is very small and the work thereby may be reduced by a factor of 2.

Further one can set up relations for other sources than point sources in analogy with (2.6). Most common are the build-up factors for isotropic point sources and for isotropic or monodirectional plane sources.

The introduction of the build-up factor B evidently does not solve the problem of finding an expression for the total flux Φ. We still have to find an expression for B, and that is a problem to which there are no general solutions, hardly even solutions within a very restricted range of parameters.

The advantage in expressing the total flux Φ in the very simple form (2.6) is, that once the B's are known numerically, (2.6) is very convenient to use. The numerical evaluation of the B's has been going on for a long time now on the largest computing machines, using Monte Carlo methods and moments methods (cf. Goldstein, 1959), and a thorough compilation on the accumulated data can be found in Goldstein's book.

In the same way as the uncollided flux from extended sources is found by integrating point kernels (2.5) over the geometrical extent of the source, the total flux Φ_V of extended sources is found by integrating the expression (2.6):

$$\Phi_V = \int_V B\Phi_u \, dv \tag{2.7}$$

over the extent v of the source. Within the range of integration one is therefore compelled to find analytical expressions for B. They are always empirical, and have to be based on existing numerical data.

It may sometimes be quite difficult to find useful analytical representations for B. In some cases the form

$$B = 1 + b \tag{2.8}$$

represents a good fit to the numerical data, particularly for small values of b. When b>1, a better fit would be

$$B = b, \tag{2.9}$$

which certainly always is incorrect for b<1.

For manual calculations in one medium a very useful form has been suggested by Taylor (1954)

$$B = A \exp(-\alpha_1 b) + (1-A) \exp(-\alpha_2 b) \tag{2.10}$$

which contains three parameters A, α_1 and α_2 independent of b but, of course, dependent on source energy E_o, material, source geometry, etc. The advantage of this form is, that it leaves the point kernel (2.5) unchanged, as can easily be shown by direct substitution of (2.5) and (2.10) into (2.7):

$$\Phi_V = \frac{S}{4\pi} \int_V \left[A \exp(-\alpha_1 b) + (1-A) \exp(-\alpha_2 b) \right] \frac{e^{-b}}{r^2} \, dv =$$

$$= \frac{S}{4\pi} \left[A \int_V \frac{e^{-b'}}{r^2} \, dv + (1-A) \int_V \frac{e^{-b''}}{r^2} \, dv \right], \tag{2.11}$$

where $b' = (1 + \alpha_1)\mu r$, and
$b'' = (1 + \alpha_2)\mu r$.

This holds evidently only for one medium. The situation is much more complicated for several media, but this problem lays outside the scope of the present work.

3. CORE APPROXIMATION 1: HOMOGENEOUS, UNIFORM SOURCE

In the present and the following chapters we want to compute the gamma ray dose rate at the cylindrical surface of the R3b reactor core. The simplest approach then is to assume the core to be a homogeneous mixture of fuel and moderator of density 1.665 g/cm^3, as given in table 1.

The axial cosine-distribution of the source strength S_v cannot easily be accounted for in manual calculations. Since we are interested in the dose rate at a point on the "equator" of the cylindrical core, where the axial distribution has its maximum, we choose the axial distribution constant, equal to its value in the equatorial plane.

The simplest approximation for the radial J_o-distribution is also a constant, which equals the average value over a radius rather than the maximum value at the center of the core. Thus the core constitutes a cylindrical source with uniform strength

$$S_v = f_a \bar{S}_v \tag{3.1}$$

from the definitions (2.1) and (2.3).

Following a suggestion by Glasstone (1955) we simplify the geometry by substituting a semi-infinite half-space for the finite cylindrical core. This is permissible when the dimensions of the cylinder are large (measured in dimensionless relaxation lengths), as shown for instance by Grotenhuis (1959), who has discussed the accuracy of the approximation in some detail.

The flux can be found by integrating (2.6) over the semi-infinite half-space in a spherical coordinate system, centered at the detector: When Φ_u is given by (2.5) and B is the point isotropic dose build-up factor in Taylor's form, we obtain

$$\Phi = \frac{S_v}{4\pi} \sum_{i=1}^{2} A_i \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta \int_0^{\infty} \frac{e^{-\mu(1+\alpha_i)r}}{r^2} r^2 dr =$$

$$= \frac{S_v}{2\mu}\left(\frac{A_1}{1+\alpha_1} + \frac{A_2}{1+\alpha_2}\right), \qquad (3.2)$$

where

r	radial distance from the detector,
A_1, A_2 = 1-A_1,	α_1 and α_2 are build-up constants,
μ	total absorption coefficient for the homogenized core, and
S_v	is given by (3.1).

Actually the formula (3.2) is an improvement on Glasstones formula for three reasons. One of them is evident: Glasstone[1] does not use any build-up, probably because he considers it an unnecessary complication to the anyway crude formula. This is an understatement, however, because the uncollided formula of Glasstone.

$$\Phi_u = \frac{S_v}{2\mu} \qquad (3.3)$$

is not conservative and a build-up correction is well needed to increase the anyway too small values of (3.3).

The two other improvements lay in the definition of S_v. Glasstone uses $\bar{S}_v$ rather than S_v as defined by (3.1), which is also unconservative. Furthermore, in the definition of $\bar{S}_v$, equation (2.1), Glasstone in an example uses $N = N_{tot}$ instead of $N = N_{esc}$. This is conservative but erroneous, because one thereby neglects the shielding effect of the individual fuel element in which the gamma rays were born, and one overestimates the flux by a factor of 2.7 in the R3b case.

The build-up constants for a compound material like the homogenized core must be determined empirically from the build-up factor curves (B a continuous function of the distance b = μr with the energy as parameter), which in turn can be found by using the interpolation methods of Goldstein and Wilkins (1954) in the curves of their report.

The numerical values of the build-up constants used are listed in table 3.

---

1) Actually the authors of the shielding chapter in Glasstone's book are E. P. Blizard and H. Goldstein.

Table 3.

Taylor build-up constants used in the R3b calculations.

Energy (MeV)	Homogenized core			Pure D_2O		
	A_1	$-\alpha_1$	α_2	A_1	$-\alpha_1$	α_2
2	6.45	0.0755	0.0912	3.0	0.0405	0.0916
3	4.7	0.0524	0.0245	3.0	0.0647	0.0562
4	2.4	0.1169	0.0336	2.2	0.1266	0.0693
6	2.0	0.131	0.024	2.0	0.131	0.024
7	1.7	0.136	0.028	2.5	0.0911	0.0629

Inserting the numerical values of $N_{esc}(\bar{E})$ from table 2, $\bar{P}_v$ from table 1 and k and f_a into (3.1), the values of S_v together with the values of the build-up constants of the homogenized core from table 3 give, when inserted into (3.2), the expected flux values. The resulting flux values together with the corresponding dose rates are listed in table 4.

Table 4.

Fluxes and dose rates by approximation 1.

Energy group (MeV)	Flux (10^{12} MeV/cm^{2}s)	Dose rate (10^7 r/h)
0 ... 3	5.48	0.88
3 ... 5	2.11	0.27
5 ... 8	0.54	0.06
0 ... 8	(not additive)	1.21
0 ... 4	5.42	0.78
4 ... 8	1.45	0.16
0 ... 8	(not additive)	0.94

It is evident from the table that the method is sensitive to the choice of energy groups. The main reason for this is the large uncertainty in the A_1 values for low energies. If we had chosen $\bar{E}$ in the energy group 0 ... 3 MeV slightly different from 2 MeV, the resulting dose rate 0.88 · 10^7 r/h would have changed considerably.

4. CORE APPROXIMATION 2: HOMOGENEOUS, NONUNIFORM SOURCE

Approximation 1 can be refined in three basically different ways:

(1) The semi-infinite half-space geometry could be exchanged to the proper cylindrical geometry. This refinement is not expected to improve the situation very much because of the large dimensions of the cylinder, but it does complicate the calculations very markedly.

(2) The uniform source distribution could be exchanged to a nonuniform, approximately correct radial distribution. This will be called core approximation 2.

(3) The homogeneous core could be exchanged to the true heterogeneous core with distributed sources. This refinement essentially includes both of the previous steps, so that it should be introduced after approximation 2. We will call it approximation 3. At this point the manual work already becomes so formidable, that one has to consider computing machines.

Approximation 2 consists in replacing the radial J_o-distribution by a cosine distribution or, if the coordinates are reversed (it should be recognized that we still have a "slab" geometry and not a cylindrical geometry), a sine distribution. The J_o-distribution cannot be used because it gives integrals which contain products of $J_o\,(x_o - x)$ and the exponential integral $E_1\,(ax + b)$, which cannot to our knowledge be solved.

Even a sine distribution cannot exactly be used, because the integral of

$$\sin x \cdot E_1\,(ax + b)$$

gives an infinite number of terms. This is in effect to replace the sine distribution

by its Taylor expansion. Retaining only the first term the flux Φ becomes

$$\Phi = \frac{S_v}{2\mu} \sum_{i=1}^{2} \frac{A_i}{1 + \alpha_i} \left\{ \sin\left[\frac{\pi R'}{2R}\right] + \frac{\pi}{4R(1 + \alpha_i)} \cos\left[\frac{\pi R'}{2R}\right] \right\}, \qquad (4.1)$$

where $R' = R - R_o$, and R and R_o are defined in table 1.
S_v is here given by

$$S_v = f_r \, f_a \, \bar{S}_v \, , \qquad (4.2)$$

where f_r is not defined by (2.2) but rather by

$$\frac{1}{f_r} = \frac{2}{R_o^2} \int_o^{R_o} r \cos\left(\frac{\pi r}{2R}\right) dr =$$

$$= \frac{2}{d^2} (\cos d + d \sin d - 1), \qquad (4.3)$$

$$d = \frac{\pi R_o}{2R} \, .$$

Substitution of numerical values gives

$$f_r \, f_a = 2.26,$$

and the results in table 5.

Table 5.

Fluxes and dose rates by approximation 2.

Energy group (MeV)	Flux (10^{12} MeV/cm^{2}s)	Dose rate (10^7 r/h)
0 ··· 3	5.97	0.96
3 ··· 5	2.43	0.32
5 ··· 8	0.98	0.11
0 ··· 8	(not additive)	1.39
0 ··· 4	6.09	0.87
4 ··· 8	1.74	0.20
0 ··· 8	(not additive)	1.07

5. CORE APPROXIMATION 3: DISTRIBUTED LINE SOURCES IN A HOMOGENEOUS MEDIUM

There are two well known methods to transform a cylindrical source S_V into an equivalent line source S_L.

a) In the self-absorption distance method, the strength of the line source is

$$S_L = \pi r^2 S_V , \qquad (5.1)$$

where r is the radius of the element, and S_V contains the total energy release N_{tot}. The attenuation within the fuel element is accounted for by locating the line source a distance z within the fuel element. z can most conveniently be found from the graphs of Rockwell (1956).

b) In the escape probability method, (5.1) still holds, but S_V now contains N_{esc} rather than N_{tot}, so that there is no further need for attenuation within the fuel element. Geometrically the line source is located at the axis of the cylinder.

Approximation 3 has been investigated under 3 slightly different circumstances. Using the above lettering for the two methods, approximation 3 is subdivided into methods

$$3a_1,\ 3b_1 \text{ and } 3b_2 .$$

The indices stand for:

1 each fuel element is immersed into pure D_2O.

2 each fuel element is immersed into a homogeneous mixture of the D_2O and the fuel elements located between the fuel element considered and the detector point.

In method $3a_1$ a subdivision into 3 energy groups has been used, whereas in the methods $3b_1$ and $3b_2$ only two groups have been used.

The flux in method $3a_1$ is given by

$$\Phi(\bar{E}) = \frac{1}{2\pi} \sum_{i=1}^{112} \frac{S_{Li}(O)}{a_i + z_i} \sum_{j=1}^{2} A_j \, \text{seci}\left\{\left[(1 + \alpha_j)\mu a_i + \mu_{en} z_i\right] , \theta_i\right\}, \qquad (5.2)$$

where

$S_{Li}(O)$ is given by (1.5) with $N(\bar{E})$ equal to $N_{tot}(\bar{E})$,

z_i self-absorption distance of the i:th element,

a_i perpendicular distance from the surface of the i:th element to the detector point,

A_j, α_j Taylor build-up parameters for pure D_2O at the energy $\bar{E}$, table 3,

μ μ_{tot} for D_2O at the energy $\bar{E}$

μ_{en} μ_{en} for fuel element at the energy $\bar{E}$

θ_i $= \text{arc tg} \frac{H_o/2}{a_i + z_i}$

seci (b, θ) is the modified secant integral

$$\text{seci}(b, \theta) = \int_0^{\theta} \exp(-b \sec x) dx, \qquad (5.3)$$

plotted by Rockwell (1956) under the notation $f(\theta, b)$.

The flux in methods $3b_1$ and $3b_2$ is given by

$$\Phi(\bar{E}) = \frac{1}{2\pi} \sum_{i=1}^{112} \frac{S_{Li}(O)}{b_i} \sum_{j=1}^{2} A_j \, \text{seci}\left[(1 + \alpha_j)\, \mu_i \, b_i, \theta_i\right], \qquad (5.4)$$

where

$S_{Li}(O)$ is given by (1.5) with $N(\bar{E})$ equal to $N_{esc}(\bar{E})$,

b_i $= a_i + \varrho$,

ϱ radius of fuel element,

θ_i $= \text{arc tg} \frac{H_o}{2b_i}$.

In method $3b_1$ $\mu_i = \mu$ and μ, A_j and α_j are as in method $3a_1$.

Table 6.

Fluxes and dose rates by the approximations 3.

Method	Energy group (MeV)	Flux (10^{12} MeV/cm^{2}s)	Dose rate (10^7 r/h)
$3a_1$	0 ... 3	3.49	0.56
	3 ... 5	2.73	0.36
	5 ... 8	0.95	0.11
	0 ... 8	(not additive)	1.03
$3b_1$	0 ... 4	4.53	0.65
	4 ... 8	1.81	0.21
	0 ... 8	(not additive)	0.86
$3b_2$	0 ... 4	3.19	0.45
	4 ... 8	1.15	0.13
	0 ... 8	(not additive)	0.58

In method $3b_2$

μ_i μ_{tot} for the homogenized sector enclosed by the core periphery and an arc with radius a_i centered at the dose point,

A_j, α_i Taylor build-up parameters for the same medium.

Concerning the numerical values of the parameters, it can be mentioned that

$z_i = 3.0$ cm for all elements for which $\frac{a_i}{\varrho} > 10$,

$\operatorname{seci}(b, \theta_i) \approx \operatorname{seci}\left[b, \frac{\pi}{2}\right]$ for all the considered angles θ_i.

The results of the manual calculations with the expressions (5.2) and (5.4) are given in table 6.

6. CONCLUSIONS

In table 7 we collect the dose rate results for all energy groups from the tables 4, 5 and 6, to facilitate a comparison. Although none of the methods can be assumed to give an absolutely correct result, upon which a quantitative comparison could be based, several qualitative conclusions can be drawn from a relative comparison of the methods.

Table 7.

Dose rates (10^7 r/h).

$\bar{E}$ (MeV)	Method				
	1	2	$3a_1$	$3b_1$	$3b_2$
2	0.88	0.96	0.56		
3	0.78	0.87		0.65	0.45
4	0.27	0.32	0.36		
6	0.16	0.20		0.21	0.13
7	0.06	0.11	0.11		
2 group total	0.94	1.07	≈0.80	0.86	0.58
3 group total	1.21	1.39	1.03		

The dependance on the number of energy groups is shown by the total dose rates of methods 1 and 2. The 3-group results are larger than the 2-group results by a factor of 1.29. In itself this factor does not say that 3-group calculations are conservative compared to 2-group calculations; it depends largely on the choice of mean energies $\bar{E}$, and another choice could well have resulted in a factor 1. However, it shows that the division into only two groups is much too coarse. The division into three groups is maybe also too coarse, but this cannot be judged from the present results. The ultimate criterion is, that a division into n or n-1 groups should not produce significantly different results, and that the width E_{max}/n of the groups should be so small, that the choice of mean energies does not influence the results significantly.

If we divide the total (3-group) dose rate of method $3a_1$ by 1.29, we obtain $0.80 \cdot 10^7$, the probable result of method $3a_1$ if it had been performed for 2 groups. This figure can be compared with the total dose rate by method $3b_1$. Although the difference is not very significant, it seems that the escape probability method (b) is conservative compared to the self-absorption distance method (a). Since the escape probability approach anyway is more straight-forward and simpler to use, it must be recommended as the better method.

The results of approximation 1 clearly show its limitations. At high energies it gives far too low values, because the constant source distribution (3.1) underestimates sources far from the detector. At 3 MeV or lower, sources far away are of little importance, whereas the distribution (3.1) overestimates sources close to the detector. We can conclude that approximation 1 gives conservative values when there are only low-energetic sources present, as in the case of a shut down reactor, where only the long-lived fission product activity is present.

If we next turn to approximation 3, it is evident from the $3b_2$ data, that the homogenization of moderator and fuel elements has a very strong influence on the attenuation. At 6 MeV the $3b_2$ value is so unconservative that the method cannot safely be recommended. Since the difference between $3b_2$ and $3b_1$ is so significant, even at 3 MeV, where most of the dose rate is contributed by the fuel elements closest to the detector, it can be concluded that the core medium becomes too dense by the method of homogenization used in $3b_2$.

Approximation 2, finally, is in general conservative. At high energies, when a large number of fuel elements contribute significantly to the dose rate, the cosine source distribution is a good approximation to the more realistic methods $3a_1$ and $3b_1$. This is confirmed by the dose rates obtained. At low energies again, approximation 2 gives higher dose rates than approximation 3, because the cosine distribution assumption introduces sources in an otherwise sourcefree region, viz. between the detector and the closest fuel element.

It can be concluded that $3b_1$ is the best method, if one requires a method which contains a minimum of approximations and gives results which are not excessively conservative. Furthermore it is well suitable for machine programming, because the accuracy can easily be increased by increasing the number of energy

groups and by taking account of the neglected axial source strength distribution. A program based on $3b_1$ can also easily be generalized to calculate the flux at any point within the reactor core and on its surface.

A program of this kind is Garlic, which has been described in a report by the author (1959 b).

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